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An engineering method for solving nonlinear aerodynamic problems of plane and axisymmetric steady flows is proposed. The method is a modification of the Monte Carlo method.

Method. Let

$$F(x, y, \psi_x, \psi_y, \psi_{xx}, \dots) = 0 \tag{1}$$

be a nonlinear differential equation in partial derivatives that describes a plane or axisymmetric steady flow, (x, y) be a point in the region ω , and let ψ, ψ_x, \dots be the function to be determined and its derivatives.

We add to Eq. (1) the boundary conditions

$$\begin{aligned} \varphi_\alpha(x, y, \psi, \psi_x, \psi_y, \dots) \Big|_\Gamma = 0, \\ \alpha = 1, 2, \dots, m, \end{aligned} \tag{2}$$

where Γ is the boundary of region ω .

We construct the function

$$\tilde{\psi} = \tilde{\psi}(x, y, C_1, \dots, C_n), \tag{3}$$

which has continuous partial derivatives to the order of the higher derivatives in Eq. (1), satisfies conditions (2) for any values of the parameters C_1, \dots, C_n , and with the proper selection of these parameters can approximate the solution of the problem.

Finding the approximate solution

$$\Phi(x, y, \bar{C}_1, \dots, \bar{C}_n) \tag{4}$$

reduces to determining those values $\bar{C}_1, \dots, \bar{C}_n$ of the parameters for which

$$\Phi(C_1, \dots, C_n) = \frac{1}{N} \sum_{i=1}^N F^2(x_i, y_i, C_1, \dots, C_n) \tag{5}$$

takes its minimum value. Here $(x_i, y_i, i = 1, 2, \dots, N)$ is a system of points from region ω , and

$$F(x, y, C_1, \dots, C_n) = F(x, y, \tilde{\psi}_x, \tilde{\psi}_y, \tilde{\psi}_{xx}, \dots) \tag{6}$$

If we cannot construct a $\tilde{\psi}$ such that conditions (2) are satisfied precisely, we can attempt, selecting C_1, \dots, C_n , to satisfy approximately both Eq. (1) and conditions (2).

In this case, Φ must be taken as

$$\Phi(C_1, \dots, C_n) = \frac{1}{N} \sum_{i=1}^N F^2(x_i, y_i, C_1, \dots, C_n) +$$

$$+ \frac{\gamma}{N_\Gamma} \sum_{i=1}^{N_\Gamma} \sum_{\alpha=1}^m f_\alpha^2(x_{i_1}, y_{i_1}, C_1, \dots, C_n). \tag{7}$$

Here,

$$f_\alpha(x, y, C_1, \dots, C_n) = \varphi_\alpha(x, y, \tilde{\psi}, \psi_x, \dots) \tag{8}$$

$x_{i_1}, y_{i_1} (i_1 = 1, 2, \dots, N_\Gamma)$ is a system of points taken at the boundary Γ . The constant γ is selected according to the equation which it is desired to satisfy with the greater accuracy: Eq. (1) or conditions (2). If approximation (4) is successfully chosen, the value of γ will have little effect on the final result.

Finding the minimum of Φ . We call an arbitrary set of values C_1, \dots, C_n a point in an n -dimensional space E^n . Each such point from region $\Omega \subseteq E^n$ corresponds to a particular Φ value. To find $\bar{C}_1, \dots, \bar{C}_n$ we use the statistical-tests method in the following form. Let the value Φ' of function Φ at some initial point C'_1, \dots, C'_n be found. Then, with the aid of a set of pseudorandom numbers ξ that are uniformly distributed in the interval $(-1, +1)$, we select a new point from the vicinity of the initial point

$$C_j = C'_j + \xi \delta C_j, \quad j = 1, 2, \dots, n. \tag{9}$$

The values δC_j determine the dimensions of this vicinity. The value of Φ at the new point is compared with Φ' . If $\Phi < \Phi'$, the new point is made the initial point for the next test. Otherwise, the initial point remains the same. As a result of this computational process, the sequence of initial points converges on point $\bar{C}_1, \dots, \bar{C}_n$. As the region of minimum Φ values is approached, the "search step" (δC_j) should be reduced. This substantially increases the efficiency of the process.

Program. The method is programmed as follows. Let us explain the designations in the flow chart (Fig. 1).

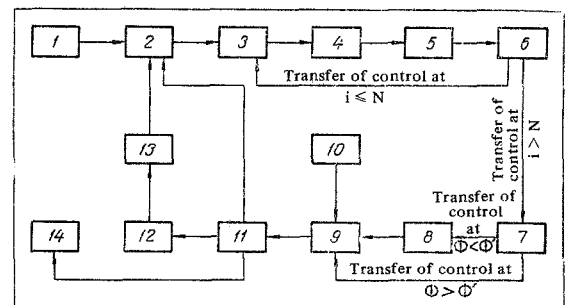


Fig. 1. Flow chart.

- 1—input to working storage and decimal-to-binary conversion.
- 2—reading of coordinates x_1, y_1 of first point from region ω .
- 3—calculation of function $\tilde{\psi}$ and its derivatives from given x_1, y_1 , and C_1, \dots, C_N . Analytic expressions as well as difference schemes can be used in calculating the derivatives. The latter method usually results in considerable shortening of the program.
- 4—calculation of $F(x_1, y_1, C_1, \dots, C_N)$ and storage of the sum ΣF^2 .
- 5—generation of coordinates x_i, y_i of next point from region ω .
- 6—transfer of control, as shown in chart, according to i -value.
- 7—comparison of obtained Φ with minimum Φ' found in previous tests. Transfer of control depends on result of this comparison.
- 8—entry of Φ, C_1, \dots, C_N in cells where Φ', C'_1, \dots, C'_N were previously stored.
- 9—calculation of new C_1, \dots, C_N by formulas (9).
- 10—generation of pseudorandom numbers ζ .
- 11—control from panel.
- 12—"search step" (δC_j).
- 13—check print.
- 14—printout.

Test of method. The proposed method was tested on the following nonlinear problems: 1) transonic flow around a cylindrical body; 2) flow of a viscous incompressible fluid between a system of staggered pipes at close to the critical Re number; and 3) transonic flow through a blade network.

The calculations were made at the Computer Laboratory of the Ivanovo Power Institute on a Ural-2 digital computer. The available data (experimental and those found by other methods) on these problems allowed the solutions to be compared with actual flows.

Take, as an example, the results for the first problem.

Flow in the vicinity of a cylindrical body is described by the following differential equation:

$$F = (c^2 - \psi_r^2) \psi_{rr} + r^{-2} (c^2 - r^{-2} \psi_\alpha^2) \psi_{\alpha\alpha} - 2r^{-2} \psi_r \psi_\alpha \psi_{r\alpha} + r^{-3} \psi_r \psi_\alpha^2 + r^{-1} c^2 \psi_r = 0. \quad (10)$$

Here r and α are polar coordinates, ψ is the velocity potential,

$$c^2 = \frac{k-1}{2} (U - \psi_r^2 - r^2 \psi_\alpha^2) \quad (11)$$

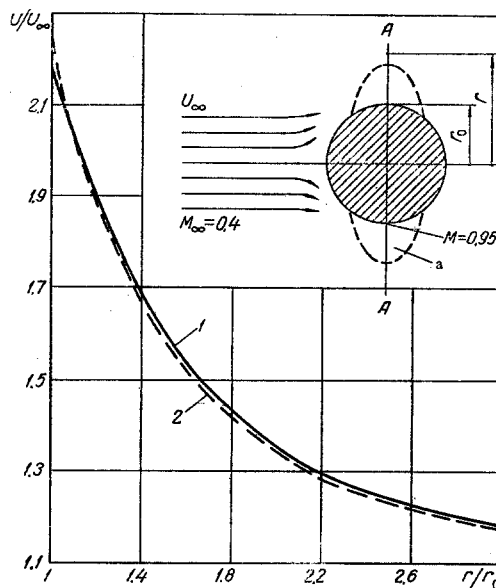


Fig. 2. Velocity distribution in section A-A: 1) experiment; 2) calculation; a) region of strong nonlinearity.

is the square of the sound velocity, and k and U are constants.

The solution is sought in the form

$$\tilde{\psi} = (r + r^{-1}) \cos \alpha + \sum_{\mu} \sum_{\nu} C_{\mu\nu} R_{\mu}(r) \cos \nu \alpha, \quad (12)$$

where

$$R_{\mu}(r) = \frac{1}{\mu} r^{-\mu} - \frac{1}{\mu+2} r^{-(\mu+2)}. \quad (13)$$

In the calculation, we limited ourselves to the six coefficients: $C_{11}, C_{13}, C_{15}, C_{31}, C_{33},$ and C_{51} .

The optimum values of these parameters were sought in accordance with the method proposed above, and also by the method proposed in [1, 2]. The results are shown in Fig. 2.

REFERENCES

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2. F. N. Yasinskii, Izvestiya VUZ. Khimiya i khim. tekhnologiya, no. 6, 1964.

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